

Chapter 12

Probability

12.1 Sample Spaces and Probability

12.2 Independent and Dependent Events

12.3 Two-Way Tables and Probability

12.4 Probability of Disjoint and Overlapping Events

12.5 Permutations and Combinations

12.6 Binomial Distributions



12.1 Sample Spaces and Probability

Probability Experiment

- A probability experiment is an action, or trial, that has varying results.



GOOD



Outcomes

BAD



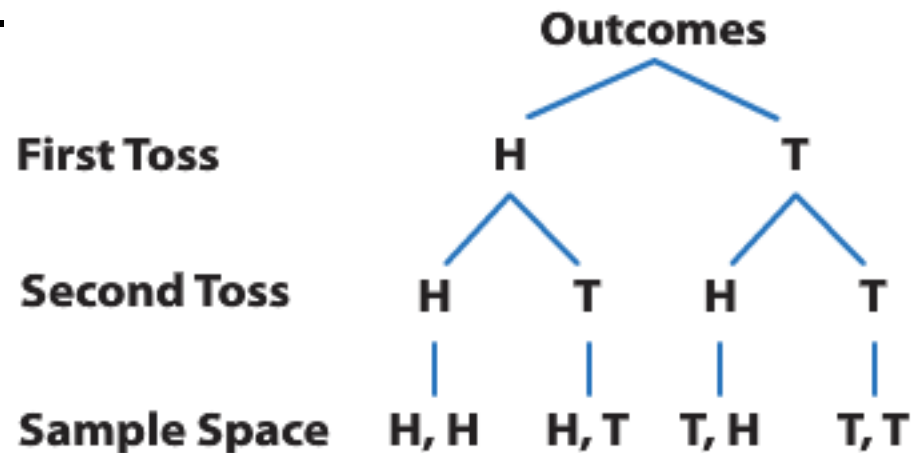
12.1 Sample Spaces and Probability

Vocabulary

- **Outcome** - The possible results of a probability experiment.
 - **Event** - A collection of one or many outcomes.
 - **Sample Space** - The set of all possible outcomes.
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Example: Flip a coin twice.













Tree Diagram →



12.1 Sample Spaces and Probability

Experiment: Roll two 6-sided dice

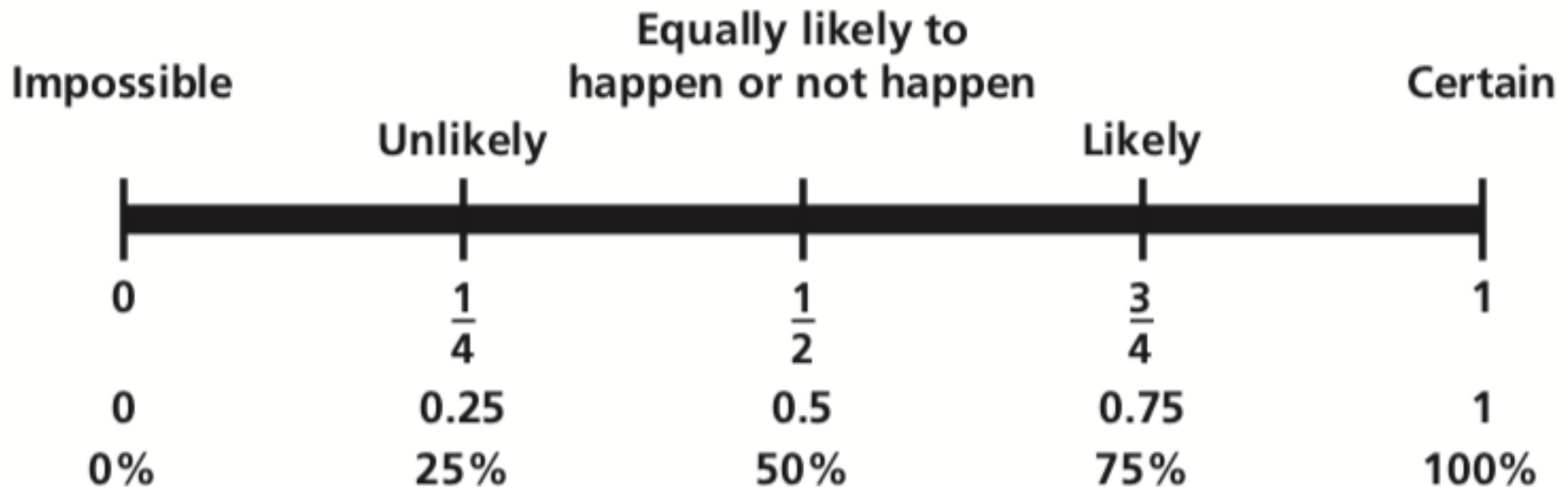
- **Sample Space** - The set of all possible outcomes: roll 1/1, or 1/2, or 1/3, ..., or 6/4, or 6/5, or 6/6
- **Event** - A set of outcomes, usually expressed as a capital letter (e.g. $A = \text{"sum of dice} = 7\text{"}$)
- **Probability of an Event** - The chance that an event "A" will happen or $P(A)$.

Possible Sums	First Die						
							
Second Die		2	3	4	5	6	7
		3	4	5	6	7	8
		4	5	6	7	8	9
		5	6	7	8	9	10
		6	7	8	9	10	11
		7	8	9	10	11	12

12.1 Sample Spaces and Probability

Probability of an Event

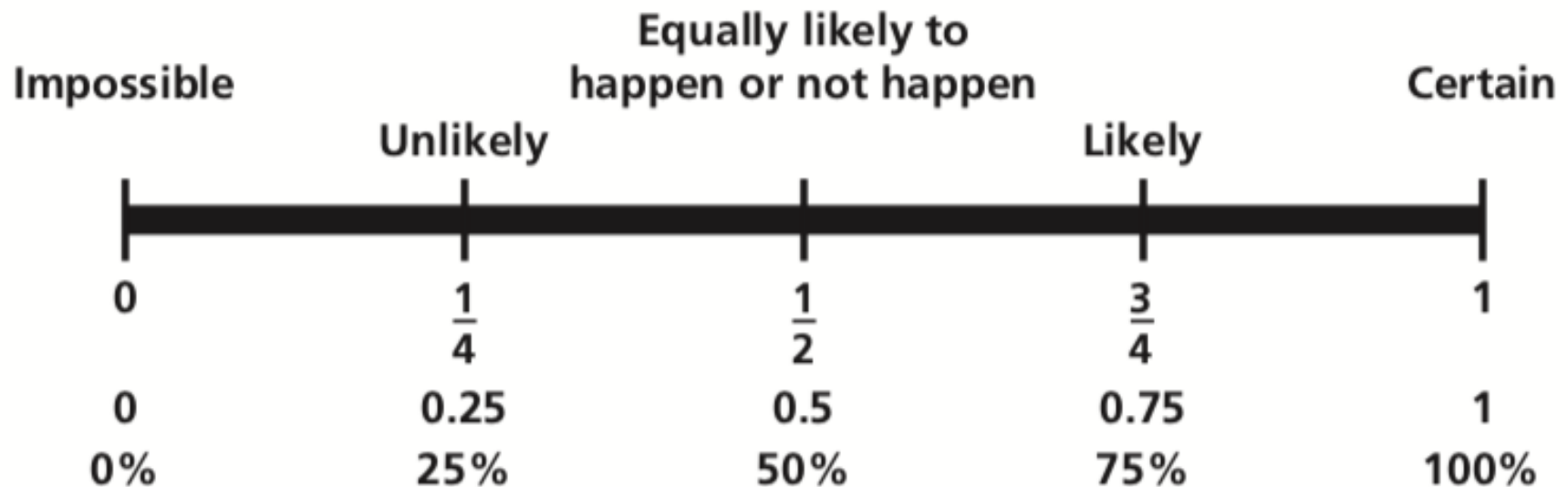
- A measure of the likelihood, or chance, that the event will occur.
- Probability is a number from 0 to 1, including 0 and 1, and can be expressed as a decimal, fraction, or percent.



12.1 Sample Spaces and Probability

Calculating Probability

$$\text{Theoretical Probability} = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$



12.1 Sample Spaces and Probability

Experiment: A student guesses on four true/false questions. What is the probability the student will make exactly two correct guesses?

- **Build a Model** - The table below represents incorrect (I) and correct (C) answers.

Number correct	Outcome
0	IIII
1	CIII ICII IICI IIIC
2	IICC ICIC ICCI CIIC CICI CCII
3	ICCC CICC CCIC CCCI
4	CCCC

exactly two correct →

$$\frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}} = \frac{6}{16} = \frac{3}{8} = 37.5\%$$

12.1 Sample Spaces and Probability

The Complement of an Event

- The probability of not event A , or $P(\bar{A})$, is computed by

$$P(\bar{A}) = 1 - P(A)$$

- For example, we found the probability of getting exactly two correct answers was $3/8$ or 37.5% .
- The probability of getting exactly zero, one, three, or four correct (not exactly two) is

$$\begin{aligned} P(\bar{A}) &= 1 - 0.375 \\ &= 0.625 = 62.5\% \end{aligned}$$

Number correct	Outcome
0	III
1	CII ICII IICI IIC
2	IICC ICIC ICCI CIIC CICI CCII
3	ICCC CICC CCIC CCCI
4	CCCC

exactly two correct →













12.1 Sample Spaces and Probability

Example: Roll two 6-sided dice

$$P(\bar{A}) = 1 - P(A)$$

Solve for the following probabilities.

- a) The sum is not 6.
- b) The sum is less than or equal to 9.

Possible Sums		First Die					
							
Second Die		2	3	4	5	6	7
		3	4	5	6	7	8
		4	5	6	7	8	9
		5	6	7	8	9	10
		6	7	8	9	10	11
		7	8	9	10	11	12

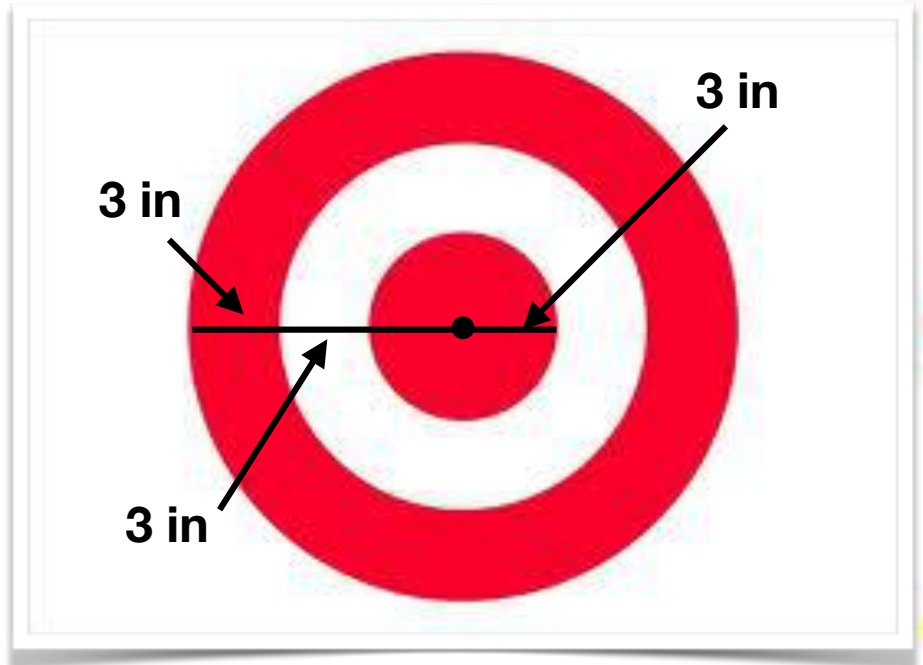
12.1 Sample Spaces and Probability

Geometric Probability

- The ratio of two lengths, areas, or volumes.

Example

- event A = hit the inside red bull's eye
- Calculate $P(A)$



12.1 Sample Spaces and Probability

Experimental Probability

- The results of repeated *trials* of a probability experiment.
- **Success** - A favorable outcome.

$$\text{Experimental Probability} = \frac{\text{Number of Successes}}{\text{Number of Trials}}$$

Example

- Repeated spins of the color spinner produced the following results.

Spinner Results			
red	green	blue	yellow
5	9	3	3

- Find experimental probabilities of the colors. e.g. P(red), etc.

Color Spinner

